# Math 2103 (2017/18)
# Assignment 4: Unitary, Raleigh and Orthopolys

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them.

1. (a) Given $\underline{v}_1 := \begin{pmatrix} 1+i \\ 1-i \\ 1-2\,i \end{pmatrix}$ and $\underline{v}_2 := \begin{pmatrix} 5 \\ 7-3\,i \\ 1-i \end{pmatrix}$, use Gram-Schmidt to form an orthogonal basis for $\mathbb{C}^3$ using the complex inner product. [4]

   (b) Given these vectors, create a matrix $Q$ such that $Q^T\overline{Q} = I$, a unitary matrix. [3]

   (c) Show that, if $H$ is a Hermitian matrix such that $H\underline{v} = \lambda\underline{v}$ then we can form a unitary matrix $Q$ by starting with $\underline{v}$ such that $Q^T H Q = \begin{pmatrix} \lambda & \cdots \\ \underline{0}^T & J \end{pmatrix}$ where J's eigenvalues are the eigenvalues of $H$ other than $\underline{v}$. [4]

   (d) Now use induction on block matrices to prove that $HR = RU$ where $U$ is an upper triangular matrix with the eigenvalues of $H$ along its diagonal and $R$ is a unitary matrix related to $Q$. Feel free to investigate this for $2 \times 2$ matrices first. [5]

2. Given this matrix $M := \begin{pmatrix} -2 & 1 & 1 & -3 \\ 1 & -2 & -3 & 1 \\ 1 & -3 & 0 & 3 \\ -3 & 1 & 3 & 0 \end{pmatrix}$ and $v := \begin{pmatrix} -7 \\ 8 \\ -7 \\ 8 \end{pmatrix}$, use the fixed Raleigh quotient method for 4 steps to find the eigenvector and eigenvalue closest to 4. Why does the method in which you just utilise $M - 4I$ work when the $M - tI$ method will fail if we recalculate $t$ each time for this given starting $\underline{v}$? [5]

3. Use either matrix solution of a system of equations or Gram-Schmidt to find a set of polynomials which form an orthogonal basis for $\mathbb{P}_3$ under the inner product

$$< p(x), q(x) >:= \int_{-1}^{2} p(x) \times q(x)\ \mathrm{d}x,$$

   if we start with $\{1, 2x-1\}$. [4]