Math 2103 Assignment 1: Algebra Review

September 15th; Due September 25th

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Maple worksheets can be submitted showing the working done. Any submissions which are received d days past the deadline can receive a maximum of $20 - d$ marks for this assignment.

1. You have been given a matrix M from $\mathbb{Z}_{13}$ to deal with in this question.

   (a) Show that $\begin{pmatrix} 5 \\ 12 \\ 6 \end{pmatrix}$ is an eigenvector of M in $\mathbb{Z}_{13}$ and deduce its eigenvalue. Get the characteristic polynomial $p(\lambda) := \det(M - \lambda I)$ and use this information to find the other eigenvalues and all characterise all eigenvectors of M. [5]

   (b) Use either the adjoint or row operations method (and the aid of Maple if wanted) to find the inverse of M and check by multiplying your answer by M manually. [4]

   (c) Get the determinant of M by using a cofactor expansion, and verify that this is equal to the product of the eigenvalues of M. Identify where this number appears in the coefficients of $p(\lambda)$ and also explain the other coefficients of $p(\lambda)$ in terms of the eigenvalues of any 3×3 polynomial. Verify your answer for M. [4]

2. (a) Show, by expanding all elements, for any 2×2 matrices that $\det(AB) = \det(A)\det(B)$ indicating which properties the underlying number system needs to have. [4]

   (b) Pick one of $n = 4, 6, 8, 9, 10$ or any other non prime number (everyone in the class needs a different n, email me when you choose yours). Working in $\mathbb{Z}_n$ find two 2×2 matrices with no zero entries and with non-zero determinants whose product has determinant 0. Explain why your two matrices do not have inverses in $\mathbb{Z}_n$. [3]

Courtney:

$$M_1 := \begin{pmatrix} 6 & 1 & 5 \\ 8 & 12 & 12 \\ 4 & 9 & 0 \end{pmatrix}$$

Levi:

$$M_2 := \begin{pmatrix} 3 & 6 & 4 \\ 9 & 0 & 7 \\ 11 & 2 & 6 \end{pmatrix}$$

Marianne:

$$M_3 := \begin{pmatrix} 6 & 7 & 9 \\ 3 & 4 & 6 \\ 8 & 11 & 11 \end{pmatrix}$$

Xucheng:

$$M_4 := \begin{pmatrix} 4 & 5 & 12 \\ 1 & 8 & 8 \\ 7 & 6 & 0 \end{pmatrix}$$