Math 2103 Assignment 2: Transformations and Bases

October 2nd; due October 16th

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Maple worksheets can be submitted showing the working done. Any submissions which are received d days past the deadline can receive a maximum of $(20 - d)$ marks for this assignment.

You have selected a linear transformation T to deal with in this question.

1. Creating a random quadratic polynomial $f(x)$ without any zero coefficients, determine which matrix is $M := T(f(x))$. [1]

2. Find the kernel of T by solving a system of equations. [3]

3. Using the kernel, or otherwise, find two other polynomials with at least one zero coefficient which also transform to M under T. [2]

4. Use the kernel-image formula to predict the dimension of the image space of T and hence find a basis for the image space of T. [3]

5. Show that M is in the image space of T by finding it as a linear combination of the basis matrices and, similarly, find two matrices P and Q which are not in the image space and which are not multiples of each other. [4]

6. Categorise *all* matrices that are not in the image space starting with the space spanned by P and Q. [2]

7. Let S be the linear transformation such that:

$$S(p + iq) := (3p + 2q)x^2 + (-5p + q)x + (p + q)$$

   Evaluate the compound linear transformation $(T \circ S)(p + iq)$ and find its kernel. [2]

8. Explore other possible linear transformations from $\mathbb{C}$ to $\mathbb{P}_2$ and try to find one which has a one dimensional kernel. [3]

$$T(ax^2+bx+c) := \begin{pmatrix} 3\,a-2\,b & -4\,a-b-4\,c \\ 2\,a-5\,b-4\,c & 4\,a+b+4\,c \end{pmatrix}$$

$$T(ax^2+bx+c) := \begin{pmatrix} -2\,a+5\,b-3\,c & 2\,a-5\,b+3\,c \\ 5\,a-2\,b+6\,c & 3\,a+3\,b+3\,c \end{pmatrix}$$

$$T(ax^2+bx+c) := \begin{pmatrix} -4\,a+2\,b+c & -3\,a+2\,b \\ -a+4\,b-5\,c & -3\,a+2\,b \end{pmatrix}$$

$$T(ax^2+bx+c) := \begin{pmatrix} 3\,a-5\,b+c & 4\,a+5\,b+3\,c \\ 3\,a+2\,b+2\,c & -2\,a+b-c \end{pmatrix}$$