Math 2103 Assignment 3: Inverses and Operators

October 18th; Due October 30th

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Maple worksheets can be submitted showing the working done. Any submissions which are received d days past the deadline can receive a maximum of $(20 - d)$ marks for this assignment.

You have selected a linear operator S to deal with in this question.

1. Determine the underlying matrix M of your operator and calculate the characteristic polynomial of M. Verify that the eigenvalues of M contain one integer and a pair of complex conjugates by either factoring the polynomial or substituting the values you obtained otherwise into the polynomial. [3]

2. Use row operations to find the eigenvector of the real eigenvalue and the complex eigenvector of one of the two complex eigenvalues. Verify that the conjugate of the complex eigenvector is the third eigenvector by multiplying it by M. [4]

3. Use the eigenvectors to create all of the S-invariant spaces in $\mathbb{P}_2$ explaining why you haven't missed any. From these spaces, take U to be a 1-dimensional one and W to be a 2-dimensional one. Check which vectors are in both U and W and hence explain whether or not $U \oplus W = \mathbb{P}_2$. [5]

4. Use Maple or other software to calculate the inverse of M and hence give $S^{-1}(px^2 + qx + r)$. Check that $S \circ S^{-1}$ is the identity operator and explain why the S^{-1}-invariant spaces are the same as the S-invariant ones. [4]

5. Create a linear operator T on $\mathbb{C}$ (cannot be a multiple of one produced by anyone else in the class and can have no zeroes in its underlying matrix) which has a kernel of dimension 1 but $T \circ T$ has a kernel of dimension 2. Now try to find a general pattern for an operator (which can have zeroes in) on a space of dimension n which eventually has its composition with itself n times becoming the operator which sends everything to the zero vector. [4]

$$S(ax^2+bx+c) := (9\,a + 27\,b - 19\,c)\,x^2 + (-8\,a - 29\,b + 18\,c)\,x - 12\,a - 46\,b + 27\,c$$

$$S(ax^2+bx+c) := (-9\,a - 6\,b + 11\,c)\,x^2 + (16\,a + 9\,b - 14\,c)\,x - 6\,a - 4\,b + 9\,c$$

$$S(ax^2+bx+c) := (-10\,a - 2\,b + 13\,c)\,x^2 + (20\,a + 6\,b - 19\,c)\,x - 5\,a + b + 11\,c$$

$$S(ax^2+bx+c) := (17\,a - 18\,b - 10\,c)\,x^2 + (14\,a - 16\,b - 10\,c)\,x - 10\,a + 15\,b + 10\,c$$