Math 2103 Assignment 5: Pseudoinverses and approximation

November 27th; Due December 17th

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Maple worksheets can be submitted via email showing the working done. Any submissions which are received d days past the deadline can receive a maximum of $(20 - d)$ marks for this assignment.

You have selected a 3×2 matrix M.

1. Verify that M has rank 1 and get $A := M^T M$ and $B := MM^T$ and their eigenvalues. [2]

2. Determine the non-zero eigenvalue's eigenvector for A (call it $\underline{v}$) and deduce the other eigenvector of A. Show that $M\underline{v}$ is an eigenvector of B and get an orthogonal basis for the eigenvectors of B of eigenvalue 0. [3]

3. Use these eigenvectors to make orthogonal matrices P and Q such that $A = PDP^T$ and $B = QEQ^T$ and verify that $M = QFP^T$ for the appropriate 3×2 matrix F. Calculate PF^+Q^T and verify that your answer is the same as Maple's `MatrixInverse(M,method=pseudo);` Identify how the process goes wrong if you use $(-M\underline{v})$ in the basis for B's eigenvectors. [6]

4. Let K be a rank 1 matrix formed by multiplying $\underline{u}$ and $\underline{w}^T$. Explain why the only non-zero eigenvalue of KK^T or $K^T K$ is always going to be $(||\underline{u}|| \times ||\underline{w}||)^2$, where the norm is the standard dot product one. [2]

5. Now let $C := B - 10I = MM^T - 10I$ and we will explore ways to approximate its eigenvalues and vectors.

   (a) Starting with $\underline{v}_1 := \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix}$, evaluate the Raleigh-Ritz quotient for C with $\underline{v}_1$ and iterate two more times. [2]

   (b) Find $\underline{v}_1$ as a linear combination of the unit eigenvectors of C and hence explain the rate of convergence of the quotient in this case. [2]

   (c) Explain what will happen if you try the procedure in (a) with any vector from the eigenspace belonging to the eigenvalue -10. [1]

   (d) Now use the inverse Raleigh-Ritz method for C starting with $\underline{v}_1$ again and find how many iterations it takes until the quotient is within 0.5 of -10. Which eigenvector is being approached? [2]

$$M_1 := \begin{pmatrix} 2 & 1 \\ 6 & 3 \\ 4 & 2 \end{pmatrix}$$

$$M_2 := \begin{pmatrix} 2 & 1 \\ 4 & 2 \\ -6 & -3 \end{pmatrix}$$

$$M_3 := \begin{pmatrix} 2 & -1 \\ 6 & -3 \\ 4 & -2 \end{pmatrix}$$

$$M_4 := \begin{pmatrix} 3 & 1 \\ 6 & 2 \\ -6 & -2 \end{pmatrix}$$