

$$\begin{aligned}
 & 3x^2 - 2xy + 2xz + 2y^2 + 2z^2 \\
 &= 3\left(x^2 - \frac{2}{3}xy + \frac{2}{3}xz\right) + 2y^2 + 2z^2 \\
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 - 3\left(\frac{y-z}{3}\right)^2 + 2y^2 + 2z^2
 \end{aligned}$$

$$\begin{aligned}
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 - \frac{1}{3}(y^2 - 2yz + z^2) + 2y^2 + 2z^2 \\
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 + \frac{5y^2}{3} + \frac{2yz}{3} + \frac{5z^2}{3} \\
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 + \frac{5}{3}\left(y^2 + \frac{2}{5}yz + \frac{z^2}{5}\right) \\
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 + \frac{5}{3}\left(y + \frac{z}{5}\right)^2 - \frac{5}{3}\left(\frac{z}{5}\right)^2 + \frac{5z^2}{3} \\
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 + \frac{5}{3}\left(y + \frac{z}{5}\right)^2 - \frac{z^2}{15} + \frac{25z^2}{15} \\
 &= 3\left(x - \frac{y}{3} + \frac{z}{3}\right)^2 + \frac{5}{3}\left(y + \frac{z}{5}\right)^2 + \frac{24z^2}{15}
 \end{aligned}$$

cf Cholesky of $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$ is $\begin{matrix} \sqrt{3} & 0 & 0 \\ -\frac{1}{\sqrt{3}} & \sqrt{\frac{5}{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{5}} & \sqrt{\frac{5}{3}} \end{matrix}$

For Matrix

$$C = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

NB $\frac{24}{15} = \frac{8}{5}$