Math 2103 (Fall 2012)
Assignment 1: Algebra and Matrices

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

1. You have selected a random matrix M for this question which comes from $\mathbb{Z}_{13}$.
   (a) Use a cofactor expansion to calculate the determinant of M and verify that it is 0. [2]
   (b) Find all 3 eigenvalues of M (by factoring a determinant) and two of its eigenvectors, including the one associated with eigenvalue 0. [8]
   (c) Give the dimension of the nullspace of M and find a basis for the image space of M. [3]
   (d) Explain why any singular matrix must have eigenvalue 0 and vice versa. [3]
2. (a) Explain (using first principles) why $(A+B)C = AC+BC$ for any matrices of appropriate sizes with entries from $\mathbb{R}$ and what properties the underlying number system must have for this to hold in general. [5]
   (b) Use matrix algebra to show that the left inverse of a matrix is the same as the right inverse; that is if $AX = I$ and $YA = I$ then $X = Y = A^{-1}$. [4]

$$\begin{pmatrix} 3 & 4 & 11 \\ 2 & 1 & 2 \\ 5 & 9 & 5 \end{pmatrix} \qquad \begin{pmatrix} 12 & 10 & 4 \\ 11 & 1 & 10 \\ 5 & 3 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 11 & 2 \\ 6 & 9 & 12 \\ 9 & 7 & 1 \end{pmatrix} \qquad \begin{pmatrix} 2 & 11 & 5 \\ 1 & 9 & 5 \\ 11 & 10 & 10 \end{pmatrix}$$

$$\begin{pmatrix} 9 & 3 & 5 \\ 5 & 7 & 5 \\ 7 & 7 & 8 \end{pmatrix} \qquad \begin{pmatrix} 12 & 11 & 10 \\ 7 & 4 & 9 \\ 4 & 9 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 4 & 11 \\ 6 & 11 & 2 \\ 3 & 7 & 7 \end{pmatrix} \qquad \begin{pmatrix} 5 & 8 & 6 \\ 8 & 5 & 9 \\ 2 & 11 & 3 \end{pmatrix}$$