Math 2103 (Fall 2012)
Assignment 2: Subspaces and transformations

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

1. Given this linear transformation from $\mathbb{P}_2$ to $\mathbb{R}^3$ find the image space and kernel. [5]

$$S(ax^2 + bx + c) := \begin{pmatrix} 2a - 2b - c \\ 3a - 2b + 2c \\ a + 2b + 10c \end{pmatrix}$$

2. Define $U + W := \{\underline{u} + \underline{w}; \underline{u} \in U, \underline{w} \in W\}$ for *any* two subspaces of *any* vector space V.
   (a) Explain why all three subspace axioms will hold for the set $U + W$. Why is $U \cap W$ a subspace of $U + W$? [4]
   (b) By considering basis sets for the various spaces, or otherwise, explain why [3]
   $$\dim(U) + \dim(W) = \dim(U + W) + \dim(U \cap W)$$
   (c) Give an example of a pair of subspaces U and W of a vector space V (everyone in the class must choose a different V) such that $U \cup W$ is not a subspace of V. Under what circumstances will it be a subspace? Verify that $U \cap W$ is a subspace of your V and determine its dimension. [4]

3. Define vector addition and multiplication as follows for the set of $n \times n$ invertible matrices:
   $$A + B := AB^{-1} \;, \quad \alpha \times A := A^{-1}$$
   (a) Explain why and how axioms A1 and A5 are true but find a counterexample to prove A3 is false. [6]
   (b) Evaluate both sides of one of the axioms S2, S3 or S4 (no more than 3 students may do any one of these, please choose a different one from your friends) and explain what families of matrices they would be true for. [3]