Math 2103 (Fall 2012)
Assignment 3: Linear Operators

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a, b and c should be replaced by the last three digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20015270 then i would take $a = 2$, $b = 7$ and $c = 10$.

1. Under what circumstances is S^k is a linear transformation (k an non-negative integer, $S^k = S(S(\ldots))$ if S is a linear transformation? What would S^0 be? Why? [3]

2. (a) Explain what rules from calculus mean that differentiation satisfies the axioms for a linear operator on the space of differentiable functions and find the kernel of this operator, D. [3]

   (b) Give an example function $f(x)$ (unique within the class) for which $D^k(f(x))$ never equals the zero function for any k. [1]

3. Given $\underline{u} := \begin{pmatrix} a & b \\ c & 0 \end{pmatrix}$, and using a well chosen basis set, attempt to create a linear operator T on the space of 2×2 matrices such that $T^3(\underline{u}) = \underline{0}$ but $T^2(\underline{u}) \neq \underline{0}$. [5]

   If you don't get a T which works out, use your best attempt for the following questions

4. Evaluate the transformations T^2 and T^3 and hence, or otherwise, find any vector $\underline{v}$ which doesn't satisfy $T^3(\underline{v}) = \underline{0}$? [4]

5. Identify at least one T-invariant subspace of each possible dimension, and ensure you have all the T-invariant subspaces of dimension 1 by getting the eigenvalues of the matrix underlying T (using Maple if necessary). [7]

6. Is T a reducible operator? [2]