Math 226 Assignment 1: Review of Concepts and Proofs

October 6, 2005

Answer all questions in any order, giving working or reasoning for all steps.

1. Explain why there is no solution to $x^2 = 18 \equiv -1$ in $\mathbb{Z}_{19}$. [2 marks]

2. Evaluate A^2 and A^4 given this matrix in $\mathbb{Z}_{19}$ where i is the square root of -1. [4]

$$A := \begin{pmatrix} -2 + 11\,i & 12 - 4\,i \\ 8 + 6\,i & -4 - 5\,i \end{pmatrix}$$

3. Find the eigenvectors and eigenvalues of A, A^2, and A^4. [6]

4. Prove, for $k = 2$ and then for any $k \geq 0$ using induction, that for any square matrix B the eigenvalues of B^k are λ_i^k when B has eigenvalues λ_i and the eigenvectors are the same as B's eigenvectors. [6]

5. Under what circumstances does the proof fail and how if we have $k = -1$ in the statement above? [2]