Math 226 Test 2: Vector Spaces and Independence

October 24, 2005

Answer all questions in any order, giving working or reasoning for all steps.

1. (a) Prove that these polynomial vectors are not linearly independent [6 marks]

$$w_1 := 1 - x + x^2 + x^3 \quad , \quad w_2 := 1 + x - x^2 - x^3 - x^4 \ ,$$
$$w_3 := 2 - x + 2x^2 - 2x^3 - 2x^4 \quad , \quad w_4 := x - 4x^3 - 2x^4$$

   (b) Using your earlier working find a subset of three of these vectors which is independent and also a proper subset which is still not independent. [2]

   (c) What are the dimensions of the spaces spanned by the three sets in (a) and (b)? [2]

2. We attempt to define a vector space on $\mathbb{R}^2$ with the following definitions of vector addition and scalar multiplication:

$$\underline{u} := \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \ , \quad \underline{v} := \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \ , \quad \underline{u} + \underline{v} := \begin{pmatrix} u_1 - v_2 \\ u_2 - 2v_1 \end{pmatrix} \ , \quad \alpha\underline{v} := \begin{pmatrix} \alpha v_1 \\ v_2 \end{pmatrix}$$

   Three of the vector space axioms A4, A5, S2, S3 and S4 are true under these definitions. Find and prove these three axioms and produce counterexamples to disprove the other two. [10]