

Math226 Assignment 1

September 28, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown on the right of the paper. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. (a) For what complex number x is this matrix singular? [3]

$$\begin{pmatrix} x & 1 + 3i \\ 7 & 2 + i \end{pmatrix}$$

- (b) What are the eigenvectors of this matrix in \mathbb{Z}_{19} ? [6]

$$\begin{pmatrix} 12 & 9 \\ 3 & 4 \end{pmatrix}$$

2. (a) Prove, by considering the expression $(1+1)(\underline{w} + \underline{-w})$ and using axiom A3 repeatedly that $\underline{s} := \underline{-w} + \underline{w} = \underline{0}$. [6]

- (b) Use the previous result (and do not use axiom A2) to prove left cancellation, that is: $(\underline{w} + \underline{u} = \underline{w} + \underline{v}) \rightarrow \underline{u} = \underline{v}$. [5]

3. Check the axioms A2, A4, S2, S3 and S4 for vectors defined on \mathbb{R}^2 as follows: [10]

$$\begin{aligned} \underline{u} &:= \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \\ \underline{v} &:= \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \\ \underline{u} + \underline{v} &:= \begin{pmatrix} | -x_1 + x_2 | \\ | +y_1 - y_2 | \end{pmatrix} \\ \alpha \times \underline{u} &:= \begin{pmatrix} y_1 \\ x_1 \end{pmatrix} \end{aligned}$$