Math226 Assignment 2

October 12, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown on the right of the paper. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. We are given four sets of vectors as follows:

   - S_n: The $n \times n$ real matrices which satisfy $A^T = A$.
   - E_k: The polynomials of maximum degree k which satisfy $f(x) = f(-x)$.
   - T_n: The $n \times n$ matrices which are upper triangular and complex.
   - Q_n: The n dimensional real number vectors whose elements sum to zero.

   (a) Verify that they are subspaces of known vector spaces. [6]

   (b) Give standard bases for each subspace. [3]

   (c) Check that your given bases are independent and span the whole subspace. [2]

   (d) Determine the dimensions of the subspaces. [2]

   [if unsure how to proceed, check with small values of n, k for partial credit]

2. (a) Use the process used to prove the fundamental theorem of linear algebra to replace vectors in the standard basis $\{v_1, v_2, v_3\}$ by the vectors in the independent set $U := \{u_1, u_2\}$. [6]

$$v_1 := \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 := \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad v_3 := \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad u_1 := \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad u_2 := \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$$

   (b) Find an general expression for $\sum_{i=1}^{3} \alpha_i v_i$ in terms of your replacement basis and check that it works by picking a random vector and substituting it. [3]

3. Given that $\{b_1, b_2, b_3, b_4\}$ is an independent set determine whether or not the following sets of vectors are independent and give the dimension of their spanning spaces. [8]

   (a) $\{b_1 + b_2 + b_4, b_1 + b_3 + b_4, b_2 - b_3\}$

   (b) $\{2b_1 + b_3, b_2 - b_4, b_2 + 3b_3, b_4 - b_1\}$

   (c) $\{-b_1 + 2b_2 - b_4, 3b_1 - b_2 + 2b_3 - b_4, b_1 + 3b_2 + 2b_3 - 3b_4, 4b_1 - 3b_2 + 2b_3\}$