

Math226 Assignment 3

October 31, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown on the right of the paper. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. (a) Find an isomorphism R from \mathbb{P}_2 to the vector space Q_4 from assignment 2 which transforms

$$1 - 3x^2 \text{ to } \begin{pmatrix} 3 \\ -4 \\ 0 \\ 1 \end{pmatrix} \text{ and } x^2 + x - 1 \text{ is transformed to } \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}. \quad [8]$$

[ensure you have a different R from all of your classmates by contacting me to let me know your choice and I will verify that you have a unique one]

- (b) Check the kernel and image of your R and hence deduce it is indeed an isomorphism. [3]
(c) Determine what $R(3x - 2)$ is for your transformation and explain why this will always be the same for any possible R . [3]
(d) Find a transformation with $R(x^2 + x - 1)$ having the given value which is not an isomorphism. What dimensions can the image spaces of such a transformation have in general? [3]

2. Given $S\left(\begin{pmatrix} a \\ b \end{pmatrix}\right) := (b + 2a)x + (3a + b)$ and $T\left(\begin{pmatrix} c \\ d \end{pmatrix}\right) := -cx - 3c + d$.

- (a) Prove S and T are both isomorphisms. [3]
(b) Determine T^{-1} and then $S \circ T^{-1}$. [4]
(c) Prove that $T \circ S^{-1}$ is the inverse of $S \circ T^{-1}$ for any S and T and verify it for your particular transforms. [6]