Math226 Assignment 4

November 14, 2006

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown on the right of the paper. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. (a) Given $T\left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right) = \begin{pmatrix} b+a \\ 5a-c \\ c-b \end{pmatrix}$ explain why T^{-1} is an isomorphism and a linear operator. [2]

   (b) Find all dimension 1 subspaces that are T-invariant and verify that one is also T^{-1}-invariant. [6]

   (c) Prove that a T-invariant subspace will always be T^{-1}-invariant if T is an isomorphism. [2]

   (d) If $T(ax^2+bx+c) := (a+4b+4c)x^2+(3b+2c)x-4b-3c$ show that $T \circ T$ is the identity operator, so T is self-inverse. [1]

   (e) Find two operators R and S which are self-inverse but $R \circ S$ is not self-inverse. [4]

2. (a) Given $U := \{k(1-x^2); k \in \mathbb{R}\}$ and $W := \{l(2+3x^2); l \in \mathbb{R}\}$, verify that $U \bigoplus W = E_2$, the even polynomials of maximum degree 2. [3]

   (b) Given $S(a+bx^2) := b-(a+b)x^2$ prove that U is not S-invariant and that, moreover, there is no non-trivial S-invariant subspace. [4]

   (c) Prove that no non-trivial linear operator R can make both U and W R-invariant. [4]

   (d) Create a linear operator R such that U is R-invariant and then find a space Y such that $U \bigoplus Y = E_2$ and Y is R-invariant. [4]