Math 226 Assignment 1: Review of Matrix Algebra

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

All numerical work in this assignment will be done in the number field $\mathbb{Z}_{11}$. The numbers represented by a, b, c and d in the questions should be replaced by the last four digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20012705 then i would take $a = 2$, $b = 7$, $c = 10$ and $d = 5$.

1. Find the inverse of the matrix $A := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and a matrix of the form $\begin{pmatrix} a & x \\ c & d \end{pmatrix}$ which has no inverse for some x. If A didn't have an inverse find an x which makes a matrix E with an inverse, and find E's inverse instead of A's inverse.

2. (a) Create a 3×3 matrix B which contains fewer than 3 zero elements which has eigenvalues equal to your a, d and a. While doing so, by use your knowledge of diagonalisation to ensure that one of a's eigenvectors is $\underline{w} := \begin{pmatrix} b \\ c \\ 3 \end{pmatrix}$.

   (b) Show that $\underline{w}$ is contained in the subspace spanned by the solutions of $(B - aI)\underline{v} = \underline{0}$.

3. (a) Prove from first principles that matrix multiplication is associative.

   (b) A matrix A is called *self-inverse* if $A^{-1} = A$. Use matrix algebra to prove that if A is self inverse then $(A - I)(A + I) = 0$.

   (c) Find two self-inverse 2×2 matrices using elements of $\mathbb{Z}_{11}$ such that their product is not self-inverse.