# Math 226 Assignment 2: Spaces and Bases

Answer all questions and show all working, reasoning and checks. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Plagiarism will earn a score of zero.

1. Define a set of vectors by the following axioms. $d$ is the last digit of your registration number.

$$V := \{\text{real symmetric } 2\times 2 \text{ matrices}\}\ ,\quad \underline{v}_1 := \begin{pmatrix} a_1 & b_1 \\ b_1 & c_1 \end{pmatrix}\ ,\quad \underline{v}_2 := \begin{pmatrix} a_2 & b_2 \\ b_2 & c_2 \end{pmatrix}$$

$$\underline{v}_1 + \underline{v}_2 := \begin{pmatrix} a_1 \times a_2 & b_1 - db_2 \\ b_1 - db_2 & c_1 + c_2 \end{pmatrix}\ ,\quad \alpha\underline{v}_1 := \begin{pmatrix} \alpha b_1 & 0 \\ 0 & \alpha \end{pmatrix}$$

   (a) State axioms A4, A5, S2, S3 and S4 in terms of $V$ and prove they are true or false.

   (b) Use algebra to find which subsets of vectors, if any, axioms are true for.

   (c) Is there a subspace of $V$ which is a vector space with these operations?

2. (a) In $\mathbb{Z}_3$, find a basis for the space spanned by the eigenvectors of $B := \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 1 & 0 & 1 \\ 2 & 1 & 2 & 0 \end{pmatrix}$ belonging to the eigenvalue of multiplicity 2.

   (b) Is your registration number vector ($(a, b, c, d)^T$ reduced modulo 3) in the eigenspace?

   (c) Use your vector or another to extend the basis to $\{b_1, b_2, b_3\}$.

      i. Find the equation for all vectors orthogonal to this new basis.

      ii. What is the dimension of this solution space?

      iii. Are your orthogonal vectors independent to the new basis?

      iv. Prove that it is true that any vector orthogonal to a proper subspace of a space is indepedent with it if the coefficients come from the real numbers and explain in your own words what can go wrong if they are from $\mathbb{Z}_3$.