Math 226 Assignment 3: Linear Transformations and Operators

Answer all questions and show all working, reasoning and checks. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Plagiarism will earn a score of zero.

The numbers represented by a, b, c and d in the questions should be replaced by the last four digits of your registration number in that order.

1. (a) Find the kernels of these transformations from fx^2+gx+h and hence identify T as the isomorphism and S as the non-isomorphism.

$$\begin{pmatrix} 2f+4g+2h \\ f+2g+h \\ af+3g-h \end{pmatrix}, \quad \begin{pmatrix} f-g \\ g+2f \\ g+h \end{pmatrix}$$

   (b) Determine the inverse of T by giving T^{-1} of the standard basis vectors.

   (c) Find the image space of S and check that its dimension satisfies the dimension theorem.

   (d) Find the only legal product of S and T^{-1}. Is it ever possible that a product of transformations is an isomorphism if one of the transformations isn't an isomorphism?

   (e) Create an isomorphism and a non-isomorphism which transform $(x-2)(x+1)$ to $\begin{pmatrix} b \\ c \\ d \end{pmatrix}$ and check they satisfy both transformation axioms.

2. (a) Find all non-trivial R-invariant subspaces of this linear operator:

$$R(px^2+qx+r) := (16\,p-9\,q+21\,r)x^2+(3\,p+4\,q+21\,r)x+(-p+3\,q+6\,r)$$

   (b) Is R reducible?