Math 226 Assignment 4: Inner Products and Complex Matrices

Answer all questions and show all working, reasoning and checks. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. Plagiarism will earn a score of zero.

1. Define an inner product of two quadratic polynomials as follows:

$$\left\langle a_1x^2+b_1x+c_1, a_2x^2+b_2x+c_2\right\rangle := 3a_1a_2+a_1b_2-a_1c_2+b_1a_2+3b_1b_2+2b_1c_2-c_1a_2+2c_1b_2+ec_1c_2$$

   where e is the second largest of your four registration number digits.

   (a) Find all polynomials which are orthogonal to the polynomial 1 under this inner product.

   (b) Which of the polynomials from (a) are also orthogonal to $x(x-3)$?

   (c) Find the Cholesky factorisation of the underlying matrix A of the inner product and hence prove axiom I4, that $\langle \underline{v}, \underline{v}\rangle > 0$ for all non-zero $\underline{v}$.

   (d) By plotting the function which is $\det(A - xI)$ verify that all eigenvalues are positive real numbers as expected. Feel free to utilise Maple.

2. Let H be the Hermitian matrix $\begin{pmatrix} d & b+ci \\ b-ci & d \end{pmatrix}$ with b, c and d your last 4 digits as usual.

   (a) Find the eigenvalues and eigenvectors of H.

   (b) Verify that the eigenvectors are orthogonal using the complex inner product.

   (c) Deduce what the eigenvectors of the 4×4 matrix with zeros everywhere apart from the top left and bottom right corners which are both copies of H will be. If you failed to solve (a), use the 2×2 from class.

   (d) Verify the eigenvalues and create an eigenvector which is not orthogonal to your two eigenvectors with the same eigenvalue, but verify that it is still orthogonal to the other two.