Math 226 Assignment 1: Matrix Algebra

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

All numerical work in this assignment will be done in the number field $\mathbb{Z}_{13}$. The numbers represented by a, b, c and d in the questions should be replaced by the last four digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20012705 then i would take $a = 2$, $b = 7$, $c = 10$ and $d = 5$.

1. (a) Get the eigenvectors of J or explain why none exist in $\mathbb{Z}_{13}$. [4]

   (b) Explain how you used algebra to find a non-diagonal matrix of the form $B := \begin{pmatrix} e & f \\ g & d \end{pmatrix}$ (with your original d) which satisfies $B^3 = I$. [9]

   (c) Explain why there will always be some $k > 0$ for which $F^k = I$ in $\mathbb{Z}_{13}$. [2]

2. Using the laws of algebra carefully prove that $(A^T)^{-1} = (A^{-1})^T$. [5]

3. Given the set of quadratic polynomials and these definitions of $+$ and $\times$, check the axioms A2, A4, S3 and S5. For those axioms which are false, explain (using algebra) which subsets of polynomials they are actually true for. [10]

$$\begin{aligned} (a_1x^2 + b_1x + c_1) + (a_2x^2 + b_2x + c_2) &:= ((c_1 + c_2)x^2 + |a_1 - a_2|x + b_1b_2)\ , \\ \alpha \times (ax^2 + bx + c) &:= ((\alpha c)x + b) \end{aligned}$$