

Math 226 Assignment 2: Bases and Transformations

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a , b , c and d in question 1 should be replaced by the last four digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20012705 then i would take $a = 2$, $b = 7$, $c = 10$ and $d = 5$.

1. (a) Find an isomorphism T from $v_1 := \begin{pmatrix} a \\ b \\ 2 \end{pmatrix}$ to $w_1 := (x - 1)^2$ and $v_2 := \begin{pmatrix} c \\ -3 \\ d \end{pmatrix}$ to $w_2 := x - 1$ by constructing two bases. [8]
- (b) Check that the kernel of T is the trivial one and find $T^{-1}(ex^2 + fx + g)$. [7]
- (c) Create a linear transformation S which maps v_1 to w_1 and v_2 to w_2 which is not one-to-one. Check that it satisfies the axioms L1 and L2. [8]
2. (a) Give a standard basis for all 2×2 complex symmetric matrices. What is the dimension of this vector space? [2]
- (b) Prove that this set of vectors is not an independent set and find the dimension of the space spanned by them. [5]

$$\begin{aligned} u_1 &:= \begin{pmatrix} 1 & i \\ i & 2 \end{pmatrix} & , & & u_2 &:= \begin{pmatrix} 1+i & 3 \\ 3 & -1 \end{pmatrix} \\ u_3 &:= \begin{pmatrix} 1 & 2-i \\ 2-i & -2 \end{pmatrix} & , & & u_4 &:= \begin{pmatrix} 4-2i & 1-i \\ 1-i & 0 \end{pmatrix} \end{aligned}$$