Math 226 Assignment 4: Inner Products

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. The letters a, b, c and d are the last four non-zero digits in your student ID number.

1. Use Cholesky factorisation to check whether or not this inner product satisfies $\langle \underline{v}, \underline{v} \rangle > 0$ for all $\underline{v} \neq \underline{0}$, where e is the largest of a, b, c and d. [5]

$$\langle \underline{u}, \underline{v} \rangle := \underline{u}^T \begin{pmatrix} 14 & 10 & 8 \\ 10 & 8 & 4 \\ 8 & 4 & e \end{pmatrix} \underline{v}$$

2. Orthogonally diagonalise $\begin{pmatrix} d & c \\ c & a \end{pmatrix}$, checking $P^T = P^{-1}$. [7]

3. Prove that $||\alpha \underline{w}|| = |\alpha| \; ||\underline{w}||$ and determine when $||\underline{v} + \underline{w}|| = ||\underline{v}|| + ||\underline{w}||$. [5]

4. In this question we shall be using this inner product:

$$\langle f(x), g(x) \rangle := \int_0^b f(x) g(x) \mathrm{d}x$$

   (a) Find the vector orthogonal to $6x + 1$ in $\mathbb{P}_1$ using Gram-Schmidt. [3]

   (b) If U has these two vectors as a basis, identify $U^\perp$ in $\mathbb{P}_2$ and check $U^\perp \cap U = \{\underline{0}\}$. [5]