# Math 226 Assignment 5: Complex Matrices

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. The letters $a$, $b$, $c$ and $d$ are the last four non-zero digits in your student ID number in increasing order.

1. Prove, using unitary diagonalisation, that if $H$ is Hermitian then $H^*H = HH^*$ and find a simple non Hermitian matrix which satisfies this relation too. [6]

2. Find an orthogonal basis for the space spanned by this set of vectors by using the Gram-Schmidt method and the complex inner product, proving why you have to use the method in one particular order now: [8]

$$\left\{ \begin{pmatrix} 1-i \\ i \\ -1+i \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ i \\ 0 \\ a+ib \end{pmatrix}, \begin{pmatrix} 2i \\ 0 \\ 1+i \\ 0 \end{pmatrix} \right\}$$

3. Verify Cayley-Hamilton for $\begin{pmatrix} d & 1+2i \\ 2-i & b+ci \end{pmatrix}$ in $\mathbb{C}$ and $\begin{pmatrix} b & 0 & d \\ 0 & 3 & c \\ 1 & -1 & 0 \end{pmatrix}$ using $\mathbb{Z}_{13}$. [5]

4. Use your knowledge of diagonalisation and eigenanalysis to create a wholly real $2 \times 2$ matrix with eigenvalue $a + ic$ and a complex one (in which each of the four elements of the matrix includes an $i$ term) which has $b$ as an eigenvalue. Give a $2 \times 2$ matrix which has both of these as the eigenvalues. [6]