Math 226 Assignment 1: Vector Spaces

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. The letters a, b, c and d are the last four non-zero digits in your student ID number.

1. Working in $\mathbb{Z}_{13}$, use row operations to find the inverse of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, if possible. [3]

2. Working in the set of $n \times n$ real matrices, we can try to define vector addition and scalar multiplication as follows:

$$\underline{v} := A \ , \quad \underline{w} := B \ , \quad \underline{v} + \underline{w} := A + A^T B \ , \quad \alpha\underline{v} := \alpha A^T$$

   Prove or disprove axioms A4, A5, S2 and S3 when $n = 2$. Does n actually matter? [9]

3. Determine whether or not these sets are subspaces of $\mathbb{P}_2$. [7]

$$\{e(1 + x^2) \mid e \in \mathbb{R}\} \ , \quad \{\text{quadratic polynomials with only real roots}\} \ ,$$

$$\{f + gx \mid f, g \in \mathbb{R}, f^2 = g^2\} \ , \quad \{p + qx + sx^2 \mid p, q \in \mathbb{R}, s \geq 0\}$$

4. Prove using axioms that, in any vector space, $\alpha\underline{0} = \underline{0}$ and, by induction, for $n \geq 0$: [6]

$$\alpha(\sum_{i=1}^{n} \underline{v}_i) = \sum_{i=1}^{n}(\alpha\underline{v}_i)$$