Math 226 Assignment 3: Operators and Inner Products

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them.

1. Create a non-trivial linear operator T with a kernel containing more than the zero vector for which
$$\left\{ k \begin{pmatrix} 3 & a \\ a & -c \end{pmatrix} \mid k \in \mathbb{R} \right\}$$
is a T-invariant space in B_2 and find another non-trivial T-invariant space apart from this one, or prove none can exist. [5]

2. (a) Explain why, if U and W are subspaces of V such that $S(\underline{u}) \in W, \forall \underline{u} \in U$ and $S(\underline{w}) \in U, \forall \underline{w} \in W$ then $U + W$ will be S-invariant even if U and W aren't. [3]

   (b) Use this to create an operator S in the space of 2×2 matrices which has no 1-dimensional S-invariant subspaces, but two 2-dimensional subspaces X and Y such that their direct sum is the whole space. [7]

3. Define an inner product on polynomials such that:
$$< f, g >:= \int_1^{b+2} fg \; \mathrm{d}x$$

   (a) Starting with the standard basis for $\mathbb{P}_2$, find an orthonormal basis for the space with respect to this inner product. [3]

   (b) Using Maple if necessary, find the space orthogonal to the space spanned by $\{x^3, x+1\}$ with respect to your inner product. [5]