Math 226 Assignment 4: Theorems and Orthogonality

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them.

1. (a) Verify the Cayley Hamilton theorem for a 3×3 matrix M which contains all eight of the numbers from your registration number. [2]

   (b) For your M separate the powers of λ to create the matrices B_j such that: [4]

$$\text{adj}(M - \lambda I) = \sum_{j=0}^{n} B_j \lambda^j$$

   (c) Using the relationship from Math 115 between a matrix and its adjoint, find the link between the coefficients of the characteristic equation of a general $n \times n$ matrix A and the B_j matrices, and hence prove the Cayley Hamilton Theorem. [7]

2. Prove this for any inner product by considering the inner product of $(\underline{u} + \underline{w})$ with itself. [3]

$$\| \underline{u} + \underline{w} \| \quad \leq \quad \| \underline{u} \| + \| \underline{w} \|$$

3. Prove or disprove these statements, using that all positive definite matrices are symmetric and that A is positive definite if $\underline{v}^T A \underline{v} > 0$ for all non-zero $\underline{v}$. [11]

   (a)

   (b) All matrices with positive determinant are positive definite.

   (c) The sum of two symmetric matrices is symmetric.

   (d) The product of two symmetric matrices is symmetric.

   (e) The sum of two positive definite matrices is positive definite.

   (f) The product of two positive definite matrices is positive definite.

   (g) The sum of two orthogonal matrices is orthogonal.

   (h) The product of two orthogonal matrices is orthogonal.