Math 226 Assignment 5: Complex and non-square Matrices

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. Feel free to attach Maple worksheets.

1. (a) Unitarily diagonalise this matrix, verifying that your eigenvectors are orthogonal under the complex inner product. [8]

$$\begin{pmatrix} 6+2\,i & 2-2\,i & -2 \\ 2 & 4-2\,i & 2\,i \\ 2\,i & 2-2\,i & 4 \end{pmatrix}$$

   (b) Exlain (using algebra) why any two Hermitian matrices (H and G) with the same eigenvalues can be written as $H = QGQ^*$ for some unitary matrix Q. [2]

2. Given $B := \begin{pmatrix} 1 & 3 \\ -1 & -3 \\ 3 & 3 \end{pmatrix}$

   (a) Calculate the eigenvalues and eigenvectors of B^TB and BB^T. [5]

   (b) Check that B is the product (in the only possible order) of the normalised eigenvector matrices and the 3×2 matrix with the square roots of the shared eigenvalues on the diagonal. [3]

   (c) Explain why the eigenvalues of C^TC and CC^T are guaranteed to be real for any $m \times n$ matrix C and why at least $|m-n|$ of them will be zero in the larger matrix. [5]

   (d) Give an example of a C with more than this number of zero eigenvalues. [2]