Math 226 2011 Assignment 1: Matrix Arithmetic and Independence

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

The numbers represented by a, b and c should be replaced by the last three digits of your registration number in that order, and if the digit is 0 it is replaced by 10. For instance, if my registration number was 20015270 then i would take $a = 2$, $b = 7$ and $c = 10$.

1. We say that a matrix X is symmetric if $X^T = X$.

   (a) Use matrix alegbra rules to prove directly that if G is symmetric then G^{-1} is symmetric, if it exists. Give an example matrix which is symmetric but which has no inverse. [3]

   (b) Give an example in $\mathbb{Z}_{13}$ to show that GH does not have to be symmetric even when both G and H are. Form a matrix F from your registration number as follows

   $$F := \begin{pmatrix} a & 11 \\ b & c \end{pmatrix}$$

   Working in $\mathbb{Z}_{13}$ find 3 different matrices E which give FE as a symmetric matrix. [2]

   (c) Find the general form for a solution for E in $\mathbb{Z}_{13}$ and represent your solution as a spanning set of independent matrices, checking their independence. [5]

   (d) How many answers are there for E which are symmetric? Can a similar procedure as in (c) be done for larger matrices such as 3×3 and can solutions be guaranteed? [3]

2. (a) Find the inverse of this matrix in $\mathbb{Z}_{13}$ using row operations: [5]

   $$P := \begin{pmatrix} c-b & 12 & 1 \\ 10 & 7 & 9 \\ 11 & 8 & 1+b-c \end{pmatrix}$$

   (b) Explain what property of this matrix ensures that there will be an inverse for everyone in the class. [1]

   (c) Use the idea of eigenvector diagonalisation $F := PDP^{-1}$ to create a matrix F which has the first and third columns of P as eigenvectors with eigenvalue 7 and the other column having eigenvalue -2. Check your answer and that the two eigenvectors you get for $\lambda = 7$ from solving the homogeneous eigenvector equation are independent and the columns of P lie within the eigenspace. [6]