Math 226 2011 Assignment 4: Symmetric Matrices plus

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own.

1. (a) Find the eigenvalues of a generic 2×2 symmetric matrix A and explain from your working why, algebraically, the determinant has to be positive for A to be positive definite. [4]

   (b) Choose three different non-zero digits from your registration number and use all of them to form three different 2×2 symmetric matrices with different determinants. Choose one with a positive determinant, call it M. Create an orthogonal matrix N from the eigenvectors of M and check that N^T gives the same as the 2×2 inverse formula. [5]

   (c) Explain why any orthogonal matrix of any size will have a determinant with absolute value 1 and why the product of any two orthogonal matrices will be necessarily orthogonal too. Is it true that any 2×2 orthogonal matrix must have only two different numbers in it and an odd number of minus signs? [4]

   (d) Choose two integers p and q different from everyone else in the class and find the unitary diagonalisation of this Hermitian matrix. [4]

$$\begin{pmatrix} p & 1+i \\ 1-i & q \end{pmatrix}$$

2. Let the inner product in this question be the following:

$$< ax^2 + bx + c, dx^2 + ex + f >:= 5ad - 5ae + 3af - 5bd + 6be - 3bf + 3cd - 3ce + 5cf$$

   (a) Check that inner product axiom P4 is satisfied by this inner product. [4]

   (b) Find a set of three polynomials which are all different from everyone else in the class which are mutually orthogonal under this inner product. [4]