HMT227
MT307

UNIVERSITY OF ZIMBABWE

Bsc Honours Part II, General Part III Mathematics

Graph Theory

June 1995
Time : 2 hours

Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

A1. What are the valency sequences of the two graphs below ? Prove whether each sequence gives rise to a unique graph or not and, if not, display a non-isomorphic graph which has the same sequence. [10]

A2. Define the terms isolated vertex, circuit and terminal vertex. Prove that a finite graph which has no terminal or isolated vertices contains a circuit. Give an example of an infinite graph which contains no terminal or isolated vertex but which doesn't contain a circuit. [10]

A3. What does it mean for a graph to be described as Eulerian ? Hamiltonian ? Draw or describe four graphs on six vertices which are, respectively, both Eulerian *and* Hamiltonian, neither Eulerian *nor* Hamiltonian, Eulerian but *not* Hamiltonian and, finally, *not* Eulerian *but* Hamiltonian. [10]

A4. Draw the graph with adjacency matrix

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}.$$

Calculate its characteristic polynomial and its chromatic polynomial.

How many different ways are there to colour the graph with two colours ? three ? [10]

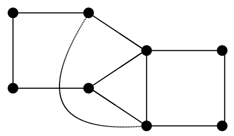
SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B8.

B5. Draw $K_{4,4}$ and the Petersen graph. Embed the former on the torus and the latter on the projective plane, clearly labeling the faces in both cases. [14]

Using these embeddings calculate the Euler-Poincaré characteristic of the two surfaces involved and determine the girths of both graphs. [6]

B6. Define the terms distance, eccentricity, radius, diameter and centre. [5]
Prove that the centre of a tree consists of either a single vertex or a pair of adjacent vertices. [10]



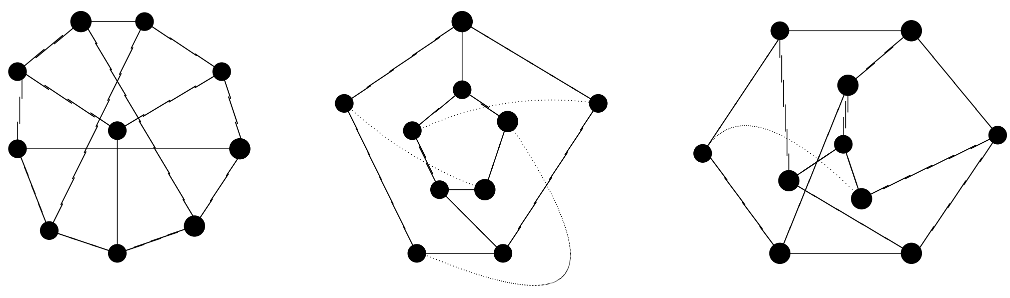
What are the eccentricities of the vertices in the graph G above ? What is the radius and diameter of G ? Clearly mark the centre of G. [5]

B7. Prove the deletion-contraction formula and deduce the addition-identification formula. [5]

Hence, or otherwise, find the chromatic polynomial of the ladder graph $L_n := P_n \times P_2$, where P_n is the path on n vertices. [7]

Find the chromatic polynomial of the graph constructed from n n-gons in a path, each joined to one other at an edge. [8]

B8. Define the graph theoretical concept of isomorphism. [2]
Identify which pairs of the graphs in the following figure are isomorphic, stating reasons or giving explicit isomorphisms where necessary. [18]



END OF QUESTION PAPER