UNIVERSITY OF ZIMBABWE

MT307

Bsc General Part III Mathematics Supplementary

GRAPH THEORY

February 1996
Time : 2 hours

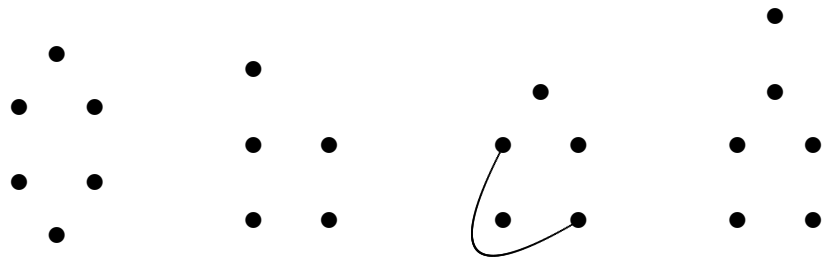
Candidates may attempt ALL questions in Section A and at most TWO questions in Section B. Each question should start on a fresh page.

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A4.

A1. How many edges and vertices does the complete bipartite graph $K_{m,n}$ have ? Embed $K_{3,4}$ in the torus and clearly indicate and count the faces in the embedding. Which face is bounded by all four vertices of the larger partite set ? Hence or otherwise embed $K_{4,4}$ and verify that the Euler-Poincaré characteristic of the torus is the same. [10]

A2. Define the graph theoretical concepts of connectivity and girth.



G_1 G_2 G_3 G_4

For each of these graphs state (giving reasons for your answer) their connectivity and their girth. [10]

A3. Using the same graphs as in question A2 determine the diameter of each graph and also give the number of vertices in each of their centres. [8]

A4. Prove that a graph is bipartite if and only if it contains no closed walk of odd length. Using this say which of the graphs in question A2 are bipartite. [12]

SECTION B (60 marks)

Candidates may attempt TWO questions being careful to number them B5 to B8.

B5. Draw these graphs: K_2, $K_{1,3}$, $K_{1,3} + K_2$, $K_{1,3} \times K_2$, $K_{1,3} \circ K_2$ and $K_2 \circ K_{1,3}$. [10]

State Kuratowski's theorem and hence or otherwise identify which of the above graphs are planar. [10]

B6. What is a self-complementary graph ? Prove that a self-complementary graph with n vertices has no vertex of valency zero or $n - 1$. [4]

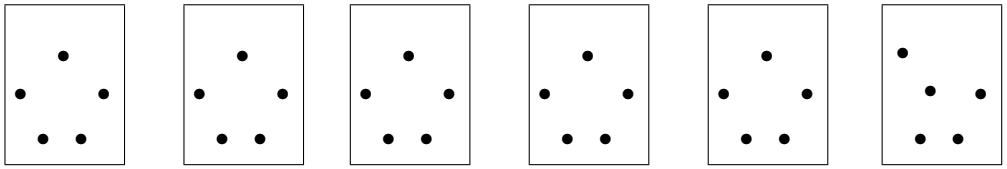
What are the feasible valency sequences for s-c graphs with five vertices ? [5]

Hence exhibit all self-complementary graphs with five vertices. [11]

B7. State the reconstruction conjecture and exhibit the small graphs which are excluded from the conjecture because they are reconstructable from each other. [4]

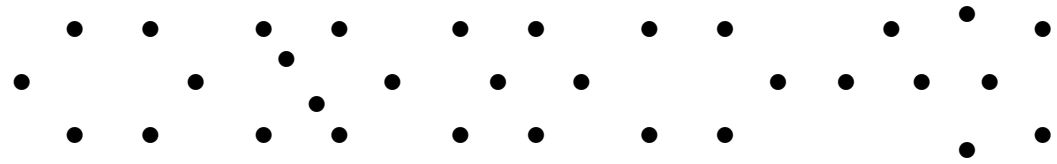
Prove Kelly's Lemma and use it to prove that the number of edges in a graph and the valency sequence of a graph is reconstructible. [11]

Reconstruct the graph G from the following deck by answering the following questions:



- What is the number of vertices in G ? [1]
- What is the maximum valency in the deck of G ? [1]
- What is the maximum valency in G and which card in the deck is associated with its removal ? [1]
- What is the valency sequence of G ? [1]
- What is G ? [1]

B8. Determine (giving reasons in all cases) whether or not any of these graphs are isomorphic to any of the others. [20]



END OF QUESTION PAPER