Math315 Assignment 1

January 17, 2007

Answer all questions and give complete reasons and checks for your answers. The parts of the questions are weighted as shown on the right of the paper. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. The game of Kaliko is played with the one-sided hexagonal 3-line tiles described in class. (`http://members.toast.net/4pf/KalikoRules.html`)
   (a) Suppose there are n colours available, determine the number of different tiles of each type. [13]
   (b) List logically all 85 tiles which can be formed with at most three colours. [5]
2. Recall that the Fibonacci numbers are defined by $f_{i+1} := f_i + f_{i-1}$ with $f_0 := 0$ and $f_1 := 1$.
   (a) Verify for $n = 0 \ldots 6$ that [2]
   $$f_{n+1} = \sum_{j=0}^{\frac{n}{2}} \binom{n-j}{j}$$
   (b) Plot the points in each sum on a North-East grid. [1]
   (c) Use the grid to formulate a combinatorial argument for why the relation is true for any n. [4]
3. (a) Find algebraically the values for r such that [2]
   $$\binom{n}{r} > \binom{n}{r-1}.$$
   (b) Prove that the rows and columns of the North-East grid are non-decreasing. [3]