Math315 Assignment 3: Combinatorics on Words

March 15, 2007

Answer all questions and give complete reasons and checks for your answers. Each question or roman numbered part of the first one is worth around the same marks. Start a fresh side of paper for each question. Hand in your rough working together with your final answers. You are reminded that plagiarism is a serious offense and if caught you will suffer the penalties specified by the University.

1. (a) We define the *complement* of a binary word w as the word $\overline{w}$ which is formed from w by replacing all 1s with 0s and vice versa. Explain why $\overline{uv} = \overline{u}\ \overline{v}$ for any binary words u and v. [1]

   (b) Prove that if f is a morphism which has $f(1) = \overline{f(0)}$ then $\overline{f(w)} = f(\overline{w})$ for any word w and verify this does not hold for the Fibonacci morphism. [4]

   (c) Define a new morphism θ as follows:

   $$\theta(0) = 01 \quad , \qquad \theta(1) = 10$$

   Find $m_i := \theta^i(0)$ for $i = 2, \ldots, 5$. [3]

   (d) Prove that all m_i have length 2^i and contain identical numbers of 0s and 1s. [2]

   (e) Using similar methods to the proof that $f_{n+1} = f_n f_{n-1}$ prove that $m_{n+1} = m_n\overline{m_n}$. [6]

   (f) By induction, or otherwise, prove that, for any positive integer k, $\widetilde{m_{2k}} = m_{2k}$ and $\widetilde{m_{2k+1}} = \overline{m_{2k+1}}$, where $\widetilde{w}$ is the reversal of w (the letters of w in reverse order). [5]

2. The following message has been received and we are using the Hamming code (7,4). Several errors have occured, but luckily we will be able to detect and correct them all. Determine the actual message that was sent and decode it: `10001001011011100110010110001001110000 1111` [5]

3. Breaking your answer to question 2 into 2 bit chunks, use Huffman to compress it and determine how many bits are saved. [if you can't get an answer to question 2, ask me for a word to compress] [4]