Math315 Assignment 1

October 2nd, 2008

Answer all questions and give complete reasons and checks for your answers. Hand in ALL of your rough working together with your final answers. The parts of the questions are weighted as shown on the right of the question. Use of Maple is encouraged where appropriate. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

1. Squarominoes are square, one-sided tiles with one colour on each of the four sides. A set of tiles is made up of one copy of each possible tile.

   (a) List logically all of the tiles which can be formed with at most four colours and identify those which just use the first two or the first three colours. [9]

   (b) Prove the general formula for the number of tiles with at most n colours. [6]

2. (a) Use the Catalan relation $C_{n+1} = \dfrac{2(2n+1)}{n+2} C_n$ to explain why all C_j are even when j is an even positive integer. Use the binomial definition of C_j and cancel some factors to find a formula for C_j which has a factor of 2^n in the numerator. [5]

   (b) Identify how many times 2 can divide into p ! and hence characterise all of the Catalan numbers which are odd. [5]