

Math 3205 Assignment 2, October 2011

October 26, 2011

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism, you can talk with others in the class but you should submit only your own work. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. (a) By expanding and manipulating the factorial forms of these binomial coefficients or using question 3 of assignment 1 show that:

$$\binom{n}{k} \times \binom{n+k}{k} = \binom{n+k}{2k} \times \binom{2k}{k}$$

and list the values of $b_n := \sum_k \binom{n}{k} \times \binom{n+k}{k}$ for n from 0 to 4. [3]

- (b) Explain why the left hand form is not suitable for use with the snake oil method, but apply snake oil to the summation of the right hand side to get the generating function for $\sum_{i \geq 0} b_i x^i$. Do *not* try to find a nice expression for b_i . [9]

2. We will use your number a from assignment 1 again here.

- (a) Use generating functions and partial fractions to find the solution for c_j given the recurrence $c_{n+1} := 4 \times c_n + 21 \times c_{n-1}$ where $c_0 := 5$ and $c_1 := a$. [6]
- (b) What is the solution for this recurrence (using generating functions)? [7]

$$e_{n+1} := 3 \times e_n + a \times n^2, \quad e_0 := 1$$