Math 3207 Assignment 1, September 2012

Clearly write your answers to the questions showing all reasons, working and checks and indicate what each mathematical calculation is doing. Do not erase anything. Include all rough work and do not commit plagiarism. Feel free to write explanations of what you are thinking at each stage, nothing you can write can lose you marks!

1. (a) Factorise your number n by using trial division, explaining each step you do in words. [1]

   (b) Use the extended Euclidean Algorithm to show that $\gcd(n, 1115) = 1$ and then reverse the algorithm to get a least linear combination of n and 1115. [5]

   (c) Use the sieve method to find the 5 primes nearest n. [4]

2. (a) Try to find integers a, b, c and d which nobody else in the class has chosen such that $d|abc$ but $d \not| ab$, $d \not| bc$ and $d \not| ac$. [2]

   (b) Prove that if an odd number divides both $x + y$ and $x - y$ then it also divides x. [4]

3. The Fibonacci numbers are defined by the recurrence $f_n := f_{n-1} + f_{n-2}$ where $f_1 = 1$ and $f_2 = 1$.

   (a) Determine the prime power factorisation of the first 16 fibonacci numbers and notice which are primes. [2]

   (b) Explain why, for all integers n, $\gcd(f_n, f_{n-1}) = 1$, using the Euclidean Algorithm. What features of the recurrence ensure this works? [4]

   (c) Pick a pair of numbers m and n which are *not* relatively prime, and verify that

   $$\gcd(f_m, f_n) = f_{\gcd(m,n)}$$

   This result is actually true for all m and n. Why does this result imply that there are an infinite number of primes? [3]