Math 3207 Assignment 1, January 2016

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. A maximum of $20 - p_y$ marks can be received for this assignment if you hand your work in y days after the deadline, where p_y is the y^{th} prime number. Let the last 4 digits of your registration number be q, r, s and t.

1. Prove that if $\gcd(a, b) = 1$ and $c \mid (a + b)$ then $\gcd(b, c) = 1$ by using least linear combinations. Find values of a, b and c all different and unique within the class such that $\gcd(a, b) \neq 1$ but the other two relations on c are both true. [3]

2. Explain why all primes greater than 4 are necessarily of the form $6j + 1$ or $6j - 1$ for some positive integer j. Mimic Theorem 2.2 in an appropriate way to give a proof that there are an infinite number of primes of the form $6j - 1$. Explain what goes wrong if you try to use this same idea to prove that there are an infinite number of primes of the form $6j + 1$. [4]

3. For these questions, work in dozenal. Let $a := 5q\delta s$ and $b := 3rt$ (this is not multiplication but treating the registration number digits as dozenal digits).

   (a) Use long multiplication to find the product of a and b. [2]

   (b) Use long division to find the quotient and remainder of a on division by b and then continue using the standard or extended Euclidean Algorithm to find the greatest common divisor of a and b. [6]

   (c) Use the sieve method, showing all details, to find the 7 primes nearest to b. [5]