

## Math 3207 Assignment 3, late October 2018

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. A maximum of  $20 - \frac{p_y}{2}$  marks can be received for this assignment if you hand your work in  $y$  days after the deadline, where  $p_y$  is the  $y^{\text{th}}$  prime number;  $p_1 := 2$ ,  $p_2 := 3$ ,  $p_3 := 5$ ,  $p_4 := 7$ ,  $p_5 := 11$ , etc.

You have randomly picked one of the slips of paper with the information on:

$n := 24149$	$n := 25789$
$k \in \{10, 11, 15, 16\}$	$k \in \{9, 12, 13, 14\}$
$b := 11$	$b := 13$

1. Given your number  $n$ , find its prime power factorisation and hence evaluate  $\tau(n)$ ,  $\phi(n)$  and  $\mu(n)$  using the formulas developed in class. Evaluate  $(\phi \star \mu)(n)$  and check that it gives the expected answer. [4]
2. Define  $\psi(n) := n \times \prod_{\text{prime } p|n} \left(\frac{1+p}{p}\right)$ .
  - (a) Evaluate  $\psi(k)$  and  $\sigma(k)$  for the four numbers given on your slip of paper. Try to guess two other numbers (bigger than 20 and not picked by anyone else in the class) for which these two numbers are different. [3]
  - (b) Prove that  $\psi$  is multiplicative by considering  $\psi(ab)$  when  $\gcd(a, b) = 1$ . [2]
  - (c) Prove that  $\psi$  is actually the same as  $J \star \mu^2$  where  $J(n) := n$ . [2]
  - (d) Determine exactly which numbers  $m$  do not satisfy  $\psi(m) = \mu(m)$  using what you've learnt. [2]
3. (a) Create a table of powers and indices using your given base  $b$  working mod 31. [2]
  - (b) Use your table of indices to find all solutions to the given equations. [5]

$$z^{13} \equiv 14 \pmod{31}, \quad 10^t \equiv 16 \pmod{31},$$

$$x^2 + 21x + 11 \equiv 0 \pmod{31}, \quad 342y^2 + 269y \equiv 423 \pmod{961},$$