# Math 3207 Assignment 4, November 2018

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You can use Maple at any point and can email me any worksheets you created.

You are reminded that plagiarism is a serious offense and when it is detected you will be punished. Feel free to discuss the questions in general with myself and your colleagues but the work attempted must be yours alone. A maximum of $20 - \frac{p_y}{2}$ marks can be received for this assignment if you hand your work in $y$ days after the deadline, where $p_y$ is the $y^{\text{th}}$ prime number; $p_1 := 2$, $p_2 := 3$, $p_3 := 5$, $p_4 := 7$, $p_5 := 11$, etc.

You have randomly picked one of the slips of paper with the information on:

1. (a) Use the Legendre symbol rules to find $\left(\frac{q}{269}\right)$ and show that $\left(\frac{23}{q}\right) = 1$. [4]

   (b) Use the first 13 squares mod $q$ to strip squares out of $x^2 \equiv 23 \pmod{q}$ until you have the solution to the equation. [3]

   (c) Let $n := \frac{5(91-q)}{4}$ for your $q$. Calculate for exactly which sorts of odd primes $t$ we have $\left(\frac{n}{t}\right) = 1$ by using the Chinese Remainder theorem to combine the separate simpler Legendre symbol factors. Give the first 10 such primes. [5]

2. (a) Carefully adapt and extend the proof of corollary 2, highlighting all differences, to show that, for prime $r$, any prime divisors of $a^r + 1$ must be of the form $2kr + 1$ for some positive integer $k$ or divisors of $a + 1$. [4]

   (b) Use (a) to factorise your $m$ as efficiently as possible, checking each feasible factor until you are sure it is fully factorised, explaining why. [4]

A4: $q := 79$ $\qquad$ $m := 11^5 + 1$

A4: $q := 83$ $\qquad$ $m := 4^{11} + 1$