Math325 Number Theory: Assignment 3 (November 4th 2010)

Answer all questions and give complete reasons and checks for your answers. Hand in ALL of your rough working together with your final answers. The parts of the questions are weighted as shown on the right of the question. Use of Maple to investigate or check answers is encouraged where appropriate but all working must be given by hand. You are reminded that plagiarism is a serious offense and when caught you will suffer the penalties specified by the University.

1. Choose a number between 20000 and 40000 different from all other students in the class which is of the form $a^p - 1$ for some odd prime p.

   (a) Give details of its prime factorisation and what factors it could feasibly have. [2]

   (b) Prove that no number of the form $a^t - 1$ is prime if t is not an odd prime. [3]

2. These numbers have been randomly assigned to you:

Student	Ben	Kristen	Ardell	Brent	Diane	Evan
k	59	43	7	31	17	10

   (a) Use repeated squaring and multiplying to verify that your number k is a primitive root mod 61. Is k a primitive root mod 171? Show there is no solution to $x^2 \equiv k \pmod{61}$. Is this true for any primitive root of any modulus? [7]

   (b) Evaluate $k^{171} \pmod{61}$ and $k^{61} \pmod{171}$. [3]

3. Let $f(n)$ and $g(n)$ be multiplicative functions.

   (a) Explain why $f(n) \times g(n)$ is a multiplicative function but $f(n) + g(n)$ is not, in general. Verify this using $f := \tau$ and $g := \phi$ with your given k and $n = 26$. [5]

   (b) Prove that $f(1) = 1$ for any non-trivial multiplicative function f. Under what circumstances would $f(g(n))$ and $\frac{f(n)}{g(n)}$ be multiplicative functions? [5]