Math 4103 (2017/18)
Assignment 1: Group Tables and Axioms

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. You will randomly pick a slip of paper with the key numbers and facts for your individualised assignment.

1. (a) For your number n, determine the 12 numbers less than n which have no divisors in common with n and derive the group table of the units in $\mathbb{Z}_n$ under multiplication mod n. [3]

   (b) Determine all of the different subgroups of your group, explaining why you are sure that you have found them all. [4]

   (c) Pick a subgroup H of cardinality 3 and create the cosets with respect to it. Redraw the group table to emphasize the cosets and hence identify which group of order 4 the factor group is isomorphic to. [3]

2. Prove the statement on your slip of paper, making sure to give all steps and explain how each line of algebra is related and what axioms or other rules are used. [3]

3. (a) You are given a set of 5 elements $\{a, b, c, d, e\}$ and are told that it is a group, there are no non-identity self-inverse elements, and that the two relations on your slip of paper are true. Stepwise, use the properties of the group table to deduce all of the remaining relations in the group. Identify the isomorphism between your group and the cyclic group generated by an element g such that $g^5 = e$ and g maps to b in the isomorphism. [5]

$\times$	e	a	b	c	d
e	e	a	b	c	d
a	a				
b	b				
c	c				
d	d				

   (b) If there was a self-inverse element other than the identity, follow the same procedure and indicate when the table loses first loses associativity and commutativity. [2]

$$n := 21 \ , \quad \forall g \in G, (g^{-1})^{-1} = g \ , \quad da = e, ab = d$$

$$n := 26 \ , \quad (gh)^2 = g^2h^2 \implies gh = hg \ , \quad bd = e, ab = c$$

$$n := 28 \ , \quad gh = hg \implies g^ih^j = h^jg^i \forall i, j \in \mathbb{Z}^+ \ , \quad ab = e, da = c$$

$$n := 36 \ , \quad g^2 = g \implies g = e \ , \quad cd = e, bc = a$$

$$n := 42 \ , \quad gh = gk \implies h = k \ , \quad db = e, ad = b$$