Math 4103 (2017/18)
Assignment 2: Isomorphism Theorems

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. You will randomly pick a slip of paper with the key numbers and facts for your individualised assignment.

1. You have selected a non-abelian group G at random.

   (a) Using the subgroup diagram of the relevant dihedral group from "Group Tables and Subgroup Diagrams" as a starting point, find all subgroups of G (giving the elements of each) and form a subgroup diagram of them. [3]

   (b) Identify the centre of G by showing which elements don't commute and the centralisers of b and $< a >$. Prove that the centre is a subgroup of any centraliser. [3]

   (c) Explain which of the subgroups are normal subgroups of G. For two (of different orders) which are not, demonstrate a conjugate subgroup. [3]

   (d) Choose non-trivial subgroups of G (none of the subgroups or their combinations are the whole group or just the identity) so that you can verify the stated isomorphism theorem for them and do so. [3]

   (e) Choose a normal subgroup K of G of order 5 or 6 and use it to create a homomorphism ϕ from G in the form of how it maps the general element of G as $\phi(a^i b^j c^k)$; check that your final answer satisfies $\phi(g_1 g_2) = \phi(g_1)\phi(g_2)$. [3]

   (f) Starting with K, demonstrate how coset enumeration could proceed until you have characterise all cosets. [2]

2. Prove that the subset of any appropriate group given on your random slip is a supgroup by verifying it is closed, includes the appropriate identity and each element has an inverse. [3]

$D_3 \oplus \mathbb{Z}_4 =< a^3 = b^2 = c^4 = e, ab = ba^2, ac = ca, bc = cb >$

Q1(d) Second Isomorphism Theorem,

Q2: Kernel of a homomorphism

$D_4 \oplus \mathbb{Z}_3 =< a^4 = b^2 = c^3 = e, ab = ba^3, ac = ca, bc = cb >$

Q1(d) Third Isomorphism Theorem,

Q2: Image of a homomorphism

$D_5 \oplus \mathbb{Z}_3 =< a^5 = b^2 = c^3 = e, ab = ba^4, ac = ca, bc = cb >$

Q1(d): Second Isomorphism Theorem,

Q2: Conjugate of a subgroup