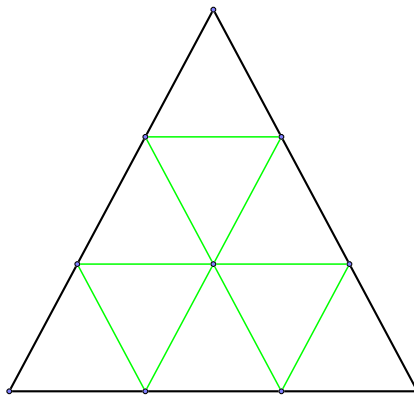


Math 4103 (2017/18)
Assignment 3: Groups and Structure

Answer all questions and show all working and check each of your results. Any rough work done before attempting your solutions should be attached to your answers as I need to know how you came up with them. You are allowed to talk with myself or other members of the class in general about the questions, but you must do them on your own. You will randomly pick a slip of paper with the key numbers and facts for your individualised assignment.

1. We will continue with a few more questions using your group G from Assignment 2.
 - (a) Create the Cayley diagram of G using a , b and c for the generator arrows using different colours. Make sure to emphasize the symmetries in your setup and drawing. [4]
 - (b) If your dihedral group was D_n , use the permutation representations $a := (1, n, (n - 1), \dots, 3, 2)$, $b := (1)(2, n)(3, (n - 1)) \dots$ and $c := ((n + 1)(n + 2) \dots)$. Determine the permutation for a^2 and verify its order from its cycle structure and check that the permutations of ab and ba^{-1} are the same, giving details of the functional way of combining the pairs of permutations and checking you get the same result as combining them from right to left. [4]
2. We are going to investigate the two-sided 3×3 equilateral triangle using D_3 as its symmetries:



- (a) Use Cauchy-Frobenius to show that there are $\binom{n^3+2}{3}$ colourings of this pattern with n colours under rotation and reflection of the triangle. [4]
- (b) For the case $n = 2$ and the range of c triangles selected, use Cauchy-Frobenius again to find the number of colourings with c small triangles coloured and then carefully list them all, using the triangular graph paper attached. [8]

$$c = 3$$

$$c = 4$$

$$c \leq 2 \text{ or } c \geq 7$$