Math415 Graph Theory: Assignment 1 (October 2007)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results.

1. (a) Find a 9 vertex self-complementary graph which is different from everyone else in the class (register your graph with me to be sure no one else claims it) [3]

   (b) Prove your graph is self-complementary by giving the isomorphism between the graph and its complement and check each edge appears in both. [5]

   (c) Prove that in any self-complementary graph on n vertices there are an even number of vertices of every valency apart from perhaps $\frac{n-1}{2}$. If you can't get that far, try to at least prove that 3 vertices of one valency are impossible. [7]

2. (a) Determine all different graphs with valency sequence (4,4,3,3,2,2). [3]

   (b) Prove all the graphs you give me are non-isomorphic and explain why, in the process you followed, none of the valency sequence have been missed. [4]

3. Define the tensor product of two graphs as follows:

$$\begin{aligned} V(G \Box H) &:= \{(g,h); g \in V(G), h \in V(H)\} \\ E(G \Box H) &:= \{(g_1,h_1)(g_2,h_2); g_1g_2 \in E(G), h_1h_2 \in E(H)\} \end{aligned}$$

   (a) Find a nice representation in terms of other products for the graph $P_2 \Box K_{1,3}$. [3]

   (b) Explain why $G \Box H$ is regular if both G and H are by determining the valency of a general vertex in the tensor product. [2]

   (c) Determine which graph is $K_2 \Box C_n$ for $n \geq 3$. [3]

4. How can you recognise and reconstruct a disconnected graph from its deck? [3]