Math415 Graph Theory: Assignment 1 (October 2009)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished.

1. (a) Completely characterise all graphs with maximum valency 2 by explaining what structure any such graph must have. [1]

    (b) Logically list all such graphs with 10 vertices and no vertices of valency 0. [6]

    (c) Explain how to recognise and reconstruct any maximum valency 2 graph from its deck. [3]

2. Find a graph with valency sequence (5,5,4,4,4,4,4,3,3) or (4,4,4,4,4,4,4,4,4) which is *not* self-complementary. Explain why there cannot be an isomorphism between your graph and its complement. [3]

   Note: your graph must be different from all of those submitted by the other members of the class, show me your graph once you find one to reserve it.

3. Prove, by induction on the number of vertices, that in any graph there is an even number of vertices of odd valency. [5]

4. Find all the different graphs with valency sequence (3,3,2,2,2,2,2) and ensure all your graphs are non-isomorphic. [7]