Math415 Graph Theory: Assignment 2 (November 2009)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished.

1. Using a modification of the tree method we used for Moore graphs, determine the maximum number of vertices possible in a r-regular *bipartite* graph of diameter d. Start by examining the cases when $d = 2$ or 3 and when $r = 2$ or 3. [8]

2. Dirac states: "For a finite graph G, if $\forall v \in V(G); \rho(v) \geq l$ then there exists a cycle of length $l + 1$ or longer in G." By considering a longest path in G, prove this. [3]

3. Using diameters, list all the trees with 6 edges and identify each tree's radius. [6]

4. Create 3 graphs, one with each of the following set of properties. Hand in all your failed attempts too, with thoughts about what went wrong each time. [6]

   - Minimum valency 2, just one cut vertex, radius 4.
   - 3-regular, 2-connected, but not 3-connected.
   - Bipartite, diameter 4, minimum valency 3.

5. Explain why a bipartite r-regular graph will always have an eigenvalue $-r$ by giving a description of an eigenvector. [2]