Math415 Graph Theory: Assignment 3 (November 2009)

Please show all working and reasoning to get full marks for any question. Hand in your rough working as well so I can see how you investigated and reached your final results. You are reminded that plagiarism is a serious offense and when it is detected you will be punished.

1. (a) Create a graph G with valency sequence (5,4,4,4,4,3,3,3) which is non-planar and show the Kuratowski subgraph proving this. [3]

   (b) Embed G on the projective plane after predicting how many faces there will be and how many faces must be triangles, at least. Count how many are in your embedding. [5]

   (c) Create a planar graph H with the same valency sequence as G, list all the faces in your embedding and check their sizes sum to the correct figure. [4]

   (d) How many colours are needed to colour your G and H? [2]

   (e) Explain why a graph with this valency sequence could never be coloured with only 2 colours. Explain what the maximum number of colours would be for a graph with this valency sequence. [4]

2. Let C be the configuration consisting of a vertex v of valency 5 with two adjacent neighbours of valency 5 and the other three neighbours of v having valency 6. Prove that C is reducible by considering its reduction to a $K_{1,4}$ by identifying 4 vertices of the boundary ring together. [7]