Math421 Group Theory: Assignment 1 January 2006

Please show all working and reasoning to get full marks for any question.

1. The group of all permutations of 6 symbols, S_6, has 720 elements.
   (a) Determine how many of each type of cycle structure there are.
   (b) Verify that there are the same number of odd and even permutations.
2. It is actually true that the two permutations $a := (12)$ and $b := (123\ldots n)$ suffice to generate all elements in S_n.
   (a) Check this statement for $n = 2$, 3 and 4 by finding all permutations in terms of products of powers of a and b.
   (b) Prove that $p^{-1}ap$ is always a 2-cycle for any permutation p.
   (c) Show how to generate all 2-cycles using just a and b in S_n and deduce that all permutations can be generated by a and b.
3. (a) Determine the elements and the table of the group which has this presentation:
   $$G := \left\langle x, y \mid x^6 = x^3y^{-2} = xyxy^{-1} = e \right\rangle$$
   (b) Determine the orders of each of the elements.
   (c) Find all cyclic subgroups in G of sizes 2, 3, 4 and 6.
   (d) Prove that no subgroup is isomorphic to S_3 and also why there can be no subgroup isomorphic to the Klein 4-group.