Math421 Group Theory: Assignment 2 February 2006

Please show all working and reasoning to get full marks for any question.

1. The presentation for D_n is $\langle a, b : a^n = b^2 = abab = e\rangle$.
   (a) Use coset enumeration with the subgroup $\langle a\rangle$ to prove that there are always $2n$ elements in D_n.
   (b) Draw the Cayley diagram for D_n using a and b for the edges.
   (c) List the elements in $G := D_5 \times \mathbb{Z}_2$ and determine the orders of each.
   (d) Use the orders to identify an a pair of elements which can map to a and b under an isomorphism between G and D_{10}, use them to generate all the elements of G and produce an identical Cayley diagram proving the isomorphism.
   (e) If $n = km$ for some odd integer k greater than 2, consider the orders of elements in D_n to show it is not isomorphic to $D_m \times \mathbb{Z}_k$ despite their orders being identical.
2. (a) Given two subgroups of G, H and K, use the subgroup test to prove that if $HK = KH$ then HK is a subgroup of G also.
   (b) If HK is a subgroup of G is it necessarily true that $HK = KH$?
   (c) Using the table for A_4 find two pairs of subgroups; one for which HK is a subgroup and one for which it is not.
   (d) What is the centre of A_n for $n = 2$, 3 and 4? For any larger n?
   (e) Determine the left and right cosets in A_4 with respect to $H := \langle(234)\rangle$ and find the normal subgroup N that A_4 contains.
   (f) Using N form the quotient group G/N and determine its group table.