

Math421 Group Theory: Assignment 1 January 2010

Please show all working and reasoning to get full marks for any question. Attach all rough work attempted to show your thought processes.

1. In any group, prove that the left identity and the right identity must be the same, using algebra. [2]

2. (a) Create the group table for the group with this presentation: [4]

$$\langle a, b, c \mid a^3 = e, b^2 = e, c^2 = e, ac = ca, ab = ba^2, bc = cb \rangle$$

- (b) Identify the orders of all elements, justifying your answers by powering. [2]

- (c) Identify all subgroups and which fundamental groups they are isomorphic to. [7]

- (d) Which subgroup of a feasible size is not in your list? Why can't it be? [1]

3. Let S be the set of $n \times n$ real valued matrices whose determinant is $+1$ or -1 .

- (a) Prove that S is a group under matrix multiplication using your knowledge of matrix algebra. [2]

- (b) Find proper, non-normal subgroups of this group of cardinality 2, 4 and ∞ . Give a different, non-trivial conjugate of each of your subgroups. [6]

- (c) A subgroup of cardinality 3 can be proved to be impossible for such 2×2 matrices. Find a subgroup of cardinality 3 for a different n . [1]